# Frequency Response of Quarter-Wave Coupled Reciprocal Stripline Junctions

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*Abstract*—**The frequency response of quarter-wave coupled reciprocal 3-port symmetrical junctions for which the reference eigennetwork appears as a short circuit at the reference terminals is presented. The equivalent circuit of such reciprocal junctions is constructed in terms of the reciprocal parts of the split admittance eigenvalues of the ideal 3-port circulator. Since the two circuits are related, the element values selected for the matching networks are the ones which apply to an ideal circulator with an overall Chebyshev response. This is done for $n=1$, 2, and 3. An important conclusion of this paper is that the design of wideband circulators is closely related to the design of wideband reciprocal 3-port junctions. The paper includes experimental results obtained on a stripline device in its magnetized and demagnetized states.**

## I. Introduction

THE FREQUENCY response of quarter-wave coupled reciprocal 3-port junctions for which the reference eigennetwork appears as a short circuit at the reference terminals is presented. Its purpose is to show that such circuits are related to a class of 3-port circulators with similar reference eigennetworks for which the splitting of the degenerate eigennetworks is symmetrical. Since the two are related, the frequency variation of one of them may be used to determine the other. Experimental correlation between the two has, in fact, been observed for a number of stripline geometries in [1]. The elements values used in this text for the matching network of the reciprocal junction are, therefore, chosen to correspond to the ones encountered in the synthesis of the ideal circulator with an overall Chebyshev response. Element values for quarter-wave coupled circulators have been given elsewhere [2]. The frequency responses of such reciprocal junctions are obtained in this paper for $n=1$, 2, and 3. The results obtained here for this class of network indicate that the frequency response of quarter-wave coupled circulators is determined by that of the unmagnetized junction in a predictable way.

The paper begins by deriving the equivalent circuit of the reciprocal junction in terms of the admittance eigenvalue of the degenerate eigennetworks of the junction. Once this equivalent circuit is known, it is possible to connect matching networks at each port and calculate the overall response in a straightforward manner. This is done for $n=1$, 2, and 3. The frequency response of such junctions is also derived at the terminals of each network. This allows the precise experimental adjustment of each matching network to obtain the desired overall response. Finally, the equivalent circuit of the ideal circulator is constructed in terms of the admittance eigennetworks of the reciprocal junction. The element values for the matching network for the reciprocal junction are therefore chosen to coincide with those of ideal circulators with Chebyshev characteristics. The text concludes with some experimental results on an $n=2$ stripline symmetrical junction in its unmagnetized and magnetized states

Manuscript received December 11, 1972; revised April 13, 1973.
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## II. Equivalent Circuit of Reciprocal 3-Port Junction

The equivalent circuit of the reciprocal junction is obtained by starting with the standard relation between the reflection coefficient $S_{11}$ and the scattering matrix eigenvalues:

$$S_{11} = \frac{s_0 + s_{+1} + s_{-1}}{3}. \tag{1}$$

The frequency dependence of the scattering matrix eigenvalues is obtained by using the relation between the scattering and admittance eigenvalues:

$$s_0 = \frac{1 - y_0}{1 + y_0} \tag{2}$$

$$s_{+1} = \frac{1 - y_{+1}}{1 + y_{+1}} \tag{3}$$

$$s_{-1} = \frac{1 - y_{-1}}{1 + y_{-1}}. \tag{4}$$

The above admittance eigenvalues are normalized to the characteristic admittance of the 50-Ω transmission lines.

For a reciprocal junction

$$s_{+1} = s_{-1} = s_1 \tag{5}$$

and

$$y_{+1} = y_{-1} = y_1. \tag{6}$$

Furthermore, the reference eigenvalue for the eigennetwork whose frequency variation is omitted is

$$s_0 = -1 \tag{7}$$

which corresponds to $S_{12} = -1$ for a circulator constructed from such eigennetworks Making use of the above relations in (1) gives

$$S_{11} = \frac{1 - 3y_1}{3 + 3y_1}. \tag{8}$$

The equivalent circuit for $S_{11}$ given by (8) is shown in Fig. 1.

The admittance eigenvalue $y_1$ has the nature of a λ/4 short-circuited transmission line.

$$y_1 = -j\left(\frac{4b_1' \cot \theta}{\pi}\right) \tag{9}$$

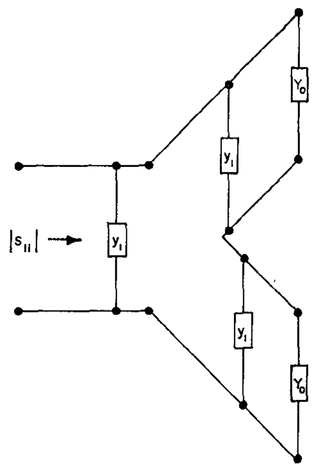


Fig. 1. Equivalent circuit of reciprocal junction in terms of eigenvalues of admittance matrix ($n=1$).

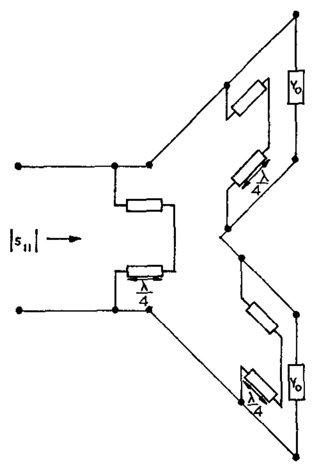


Fig. 2. Equivalent circuit of reciprocal junction in terms of quarter-wave stubs ($n=1$).

where

$$\theta = \frac{\pi}{2}(1+\delta). \tag{10}$$

Here $b_1'$ is the normalized susceptance slope parameter, and $2\delta$ is a normalized frequency variable $(\omega_0-\omega)/\omega_0$.

The equivalent network in terms of distributed $\lambda/4$ short-circuited transmission lines is shown in Fig. 2.

## III. Reflection Coefficient of Quarter-Wave Reciprocal 3-Port Junction

The equivalent circuit of the overall network is shown in Fig. 3. Each matching network is represented in terms of an overall $ABCD$ matrix. The ones studied in this paper consist of quarter-wave-long impedance transformers. The reflection coefficient of the overall network is given by straightforward calculation by

$$|\gamma| = \left[\frac{(DE-A-BF)^2+(C-BE-DF)^2}{(DE+A+BF)^2+(C+BE-DF)^2}\right]^{1/2} \tag{11}$$

$E$ and $F$ are the real and imaginary parts of the equivalent network due to ports 2 and 3 which load the $ABCD$ matrix at

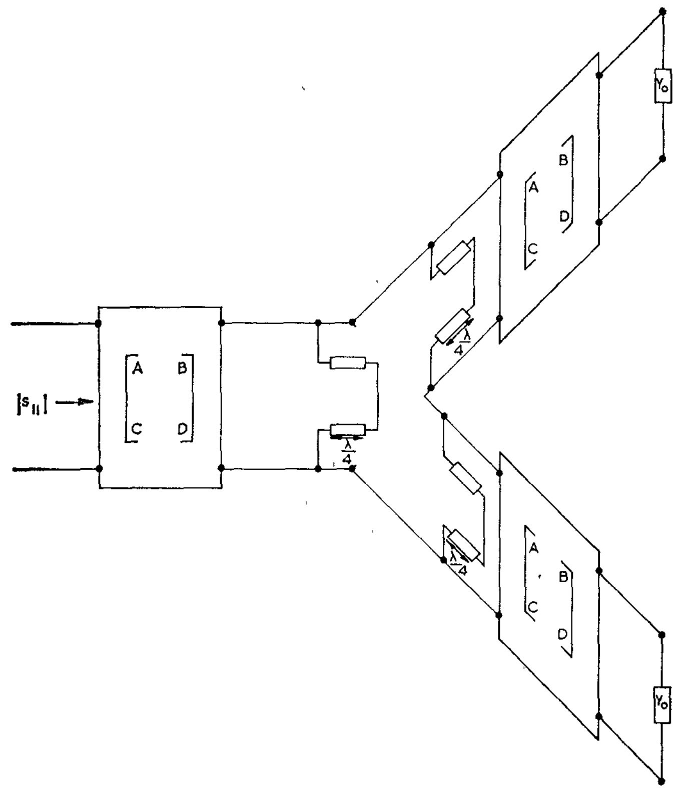


Fig. 3. Equivalent circuit of wide-band reciprocal junction in terms of $ABCD$ networks.

port 1, and $A$, $B$, $C$, and $D$ are real numbers. The VSWR $r$ is given in terms of the reflection coefficient in the usual way by

$$r = \frac{1+|\gamma|}{1-|\gamma|}. \tag{12}$$

The minimum value for the reflection coefficient is $|\gamma| = 1/3$ which corresponds with the maximum power transfer condition through the junction.

## IV. Frequency Response of $n=1$ Network

The frequency response at the junction terminals is obtained with the $ABCD$ parameters given by

$$A = \cos\theta \tag{13}$$

$$B = \sin\theta \tag{14}$$

$$C = \sin\theta \tag{15}$$

$$D = \cos\theta \tag{16}$$

and

$$E = 2 \tag{17}$$

$$F = 3\left(\frac{4b_1' \cot\theta}{\pi}\right). \tag{18}$$

The definition here for $\theta$ is the same as that given in (10). An approximate relation in the case of $n=1$ is obtained by replacing the distributed network by a lumped-element one.

$$y_1 = j2\delta b_1'. \tag{19}$$

This last equation is obtained directly from (9).

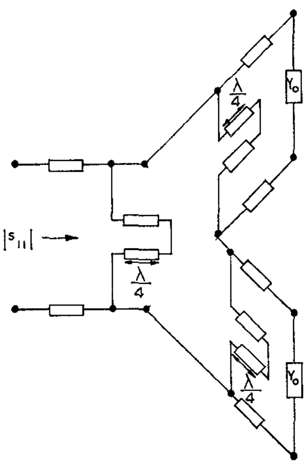

Fig. 4. Equivalent circuit of quarter-wave coupled reciprocal junction ($n=2$).

It is also observed that the normalized susceptance slope parameter is equal to the loaded $Q$-factor.

$$b_1' = \frac{B_1'}{Y_0} = Q_L. \tag{20}$$

The result in terms of the VSWR $r$ is

$$b_1' = \frac{\frac{2}{3}\left[\frac{r^2 - 2.5r + 1}{2r}\right]^{1/2}}{2\delta}. \tag{21}$$

## V. Frequency Response of $n=2$ Network

The frequency response of the $n=2$ network is obtained with

$$A = \cos\theta \tag{22}$$

$$B = \frac{\sin\theta}{y_t} \tag{23}$$

$$C = y_t \sin\theta \tag{24}$$

$$D = \cos\theta \tag{25}$$

$$E = \frac{2(AD + BC)}{A^2 + B^2} \tag{26}$$

$$F = \frac{\frac{3(4b_1' \cot\theta)}{\pi}(A^2 + B^2) - 2(AC - BD)}{A^2 + B^2} \tag{27}$$

where $y_t$ is the transformer admittance. This network is shown in Fig. 4.

## VI. Frequency Response of $n=3$ Network

The frequency response of the $n=3$ network is obtained with

$$A = \cos^2\theta - \frac{y_{02}}{y_{01}} \sin^2\theta \tag{28}$$

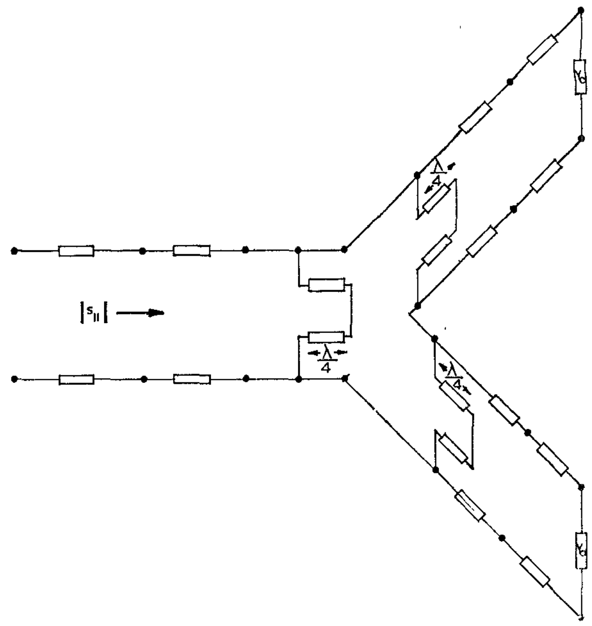

Fig. 5. Equivalent circuit of two-section quarter-wave coupled reciprocal junction ($n=3$).

$$B = \left(\frac{1}{y_{01}} + \frac{1}{y_{02}}\right) \sin\theta \cos\theta \tag{29}$$

$$C = (y_{01} + y_{02}) \sin\theta \cos\theta \tag{30}$$

$$D = \frac{-y_{01}}{y_{02}} \sin^2\theta + \cos^2\theta \tag{31}$$

$$E = \frac{2(A_1D_1 + B_1C_1)}{A_1^2 + B_1^2} \tag{32}$$

$$F = \frac{\frac{3(4b_1' \cot\theta)}{\pi}(A_1^2 + B_1^2) - 2(A_1C_1 - B_1D_1)}{A_1^2 + B_1^2} \tag{33}$$

where

$$A_1 = \cos^2\theta - \frac{y_{01}}{y_{02}} \sin^2\theta \tag{34}$$

$$B_1 = \left(\frac{1}{y_{01}} + \frac{1}{y_{02}}\right) \sin\theta \cos\theta \tag{35}$$

$$C_1 = (y_{01} + y_{02}) \sin\theta \cos\theta \tag{36}$$

$$D_1 = \frac{-y_{02}}{y_{01}} \sin^2\theta + \cos^2\theta. \tag{37}$$

Here $y_{01}$ and $y_{02}$ are the admittances of the transformers. This network is shown in Fig. 5.

## VII. Equivalent Circuit of 3-Port Junction Circulator

In order to use the matching elements encountered in the synthesis of quarter-wave coupled circulators in the design of wideband reciprocal junctions, it is necessary to relate the two equivalent circuits. To do this, the equivalent circuit of a 3-port junction circulator is developed in terms of the admittance $y_1$ encountered in the circuit of the reciprocal junction. The admittance eigenvalues of an ideal circulator in the vicinity of the circulation frequency with symmetrical splitting of the eigennetworks with the direct magnetic field is [4]

$$y_{+1} = y_1 - \frac{j}{\sqrt{3}} \tag{38}$$

$$y_{-1} = y_1 + \frac{j}{\sqrt{3}} \tag{39}$$

where $y_1$ is the admittance eigenvalue of the reciprocal junction.

The reflection coefficient $s_{+1}$ of an ideal circulator in the vicinity of the circulation frequency is now given by

$$s_{+1} = \frac{1 - \left(y_1 - \frac{j}{\sqrt{3}}\right)}{1 + \left(y_1 - \frac{j}{\sqrt{3}}\right)}. \tag{40}$$

In a similar way one has for the eigenvalue $s_{-1}$

$$s_{-1} = \frac{1 - \left(y_1 + \frac{j}{\sqrt{3}}\right)}{1 + \left(y_1 + \frac{j}{\sqrt{3}}\right)}. \tag{41}$$

The eigenvalue for which the frequency variation is omitted is again

$$s_0 = -1. \tag{42}$$

The reflection coefficient is now obtained by taking a linear combination of $s_0$, $s_{+1}$, and $s_{-1}$. The result in the vicinity of circulation frequency is

$$S_{11} \approx \frac{y_1}{2}. \tag{43}$$

The normalized input admittance for this circuit in terms of the original variables is

$$y_{\text{in}} = 1 - j\left(\frac{4b_1' \cot \theta}{\pi}\right). \tag{44}$$

An approximate equivalent network for this equation is an ideal circulator available at any frequency with an admittance $y_1$ connected at each port [4]. This is shown in Fig. 6.

Writing $S_{11}$ in terms of $r$ now gives

$$b_1' = \frac{2(r-1)}{2\delta(r+1)}. \tag{45}$$

It is seen from (21) and (45) in this paper that the two $n=1$ circuits are, indeed, constructed in terms of the same admittance eigenvalue. The frequency variation of one of these circuits may, therefore, be used to infer one or the other. Reference [1] gives some experimental results on the frequency responses of such circuits from which correlation between theory and experiment is possible. Applying (21) of this paper to [1, fig. 6] gives, for the susceptance slope parameters of the different geometries, $b_{1a}'=5.65$, $b_{1b}'=3.93$, $b_{1c}'=3.33$, and $b_{1d}'=1.84$. In a similar way, applying (45) to [1, fig. 7] gives, for the corresponding geometries, $b_{1a}'=6.6$, $b_{1b}'=3.89$,

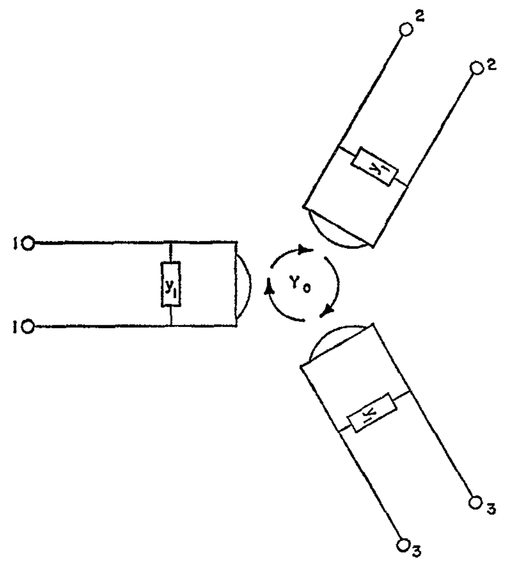


Fig. 6. Equivalent circuit of ideal circulator.

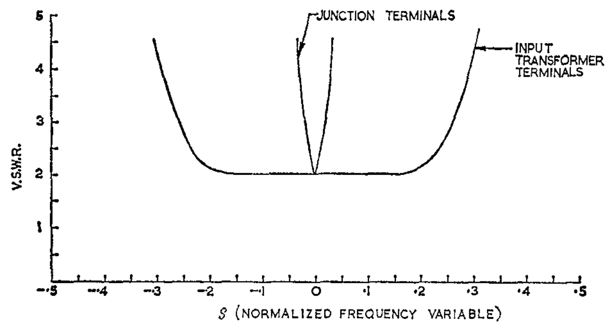


Fig. 7. Frequency response for $n=2$ junction with $r=1.15$ and $2\delta_0=0.35$ at each pair of terminals.

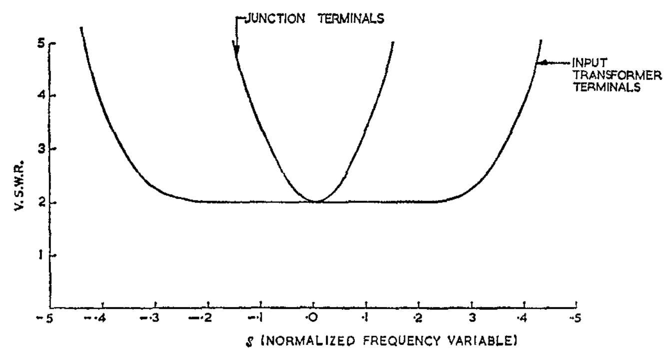


Fig. 8. Frequency response for $n=2$ junction with $r=1.15$ and $2\delta_0=0.50$ at each pair of terminals.

$b_{1c}'=3.51$, and $b_{1d}'=1.62$. A comparison of these two sets of results shows that the circuits are indeed constructed in terms of the same admittance eigenvalue.

## VIII. Computations

This section describes some computations on the frequency responses of $n=2$ and $n=3$ networks. The examples treated in this section apply to networks which when magnetized yield circulators with $r=1.15$ and $2\delta_0=0.35$, 0.50, and 0.66. The element values for the networks are given in [2]. The results are shown in Figs. 7 and 8 in the case of $n=2$, and in

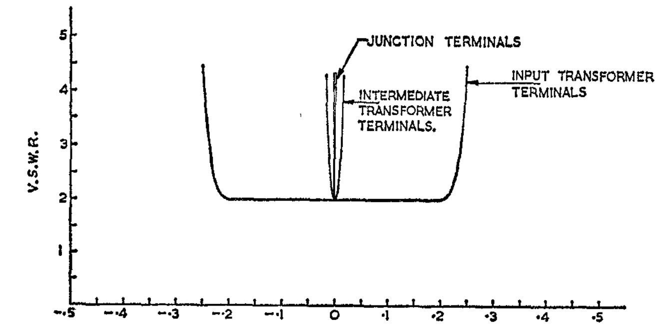


Fig. 9. Frequency response for $n=3$ junction with $r=1.15$ and $2\delta_0=0.35$ at each pair of terminals.

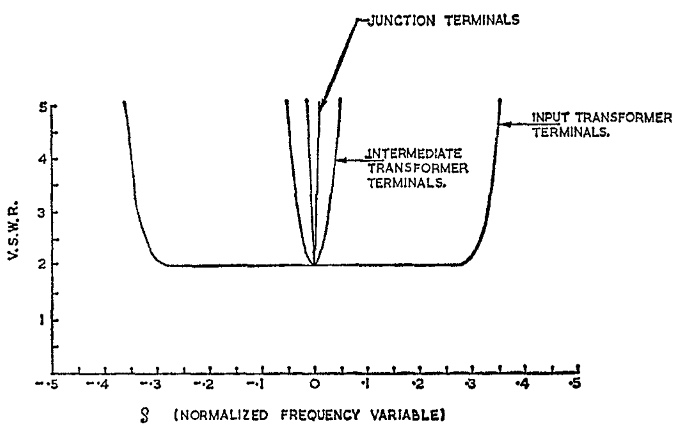


Fig. 10. Frequency response for $n=3$ junction with $r=1.15$ and $2\delta_0=0.50$ at each pair of terminals.

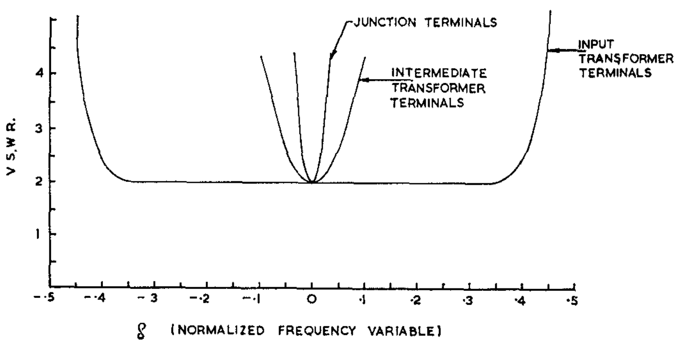


Fig. 11. Frequency response for $n=3$ junction with $r=1.15$ and $2\delta_0=0.65$ at each pair of terminals.

Figs. 9, 10, and 11 in the case of $n=3$. The frequency responses are given in each case at the terminals of each network. The frequency behavior at the intermediate transformer terminals in the case of $n=3$ is obtained using the material in Section V with $y_t$ replaced by $Y_{02}$. This allows the precise experimental adjustment of each matching network to obtain the overall response. These illustrations show that the bandwidth of the overall reciprocal network is comparable to that of the junction magnetized to form a circulator. This implies that the construction of wideband circulators is closely related to that of wideband reciprocal 3-port junctions. Here, the bandwidth of the reciprocal junction is that which coincides with maximum power transfer through the junction. It is also observed that the frequency responses of the $n=2$ and $n=3$ reciprocal

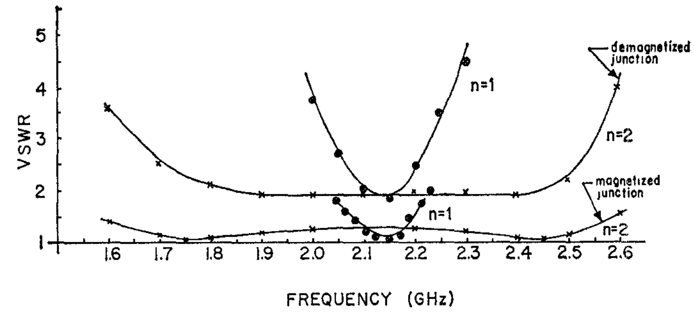


Fig. 12. Experimental frequency response of demagnetized and magnetized $n=1$ and $n=2$ stripline circuits.

junctions associated with identical circulator characteristics are similar. The only difference is that the slope at the band edges for the $n=2$ network is less than that for $n=3$.

## IX. Experimental Results

This section gives the experimental frequency response of 3-port reciprocal and nonreciprocal junctions with $n=1$ and $n=2$ networks. The configuration studied was a stripline one. The $n=1$ network consists of a 27-mm diameter garnet disk with a magnetization of 0.050 Wb/m$^2$ and a dielectric constant of $\epsilon_r=14.3$. Its unmagnetized and magnetized frequency responses are shown in Fig. 12. The susceptance slope parameter $b_1'$ of the unmagnetized junction is $b_1'=4.65$, while that of the magnetized one is $b_1'=5.7$. The dielectric constant $\epsilon_r$ of the quarter-wave impedance transformer used for the $n=2$ network was $\epsilon_r=3.2$, which corresponds to $y_t=1.79$. Such an $n=2$ network coincides with a circulator characteristic with $r=1.25$ and $2\delta_0=0.40$. The frequency responses for this network are shown in Fig. 12 also. Here the bandwidth of the magnetized junction is $2\delta_0=43$ percent at $r=1.25$. It is observed that the bandwidth between the minimum VSWR points of the magnetized junction is approximately 0.707 of that of the full bandwidth, as it should be. It is also seen that for both the $n=1$ and $n=2$ cases, the frequency behavior of the reciprocal junction may be used to predict the nonreciprocal one. The frequency response of the reciprocal $n=2$ network is slightly asymmetrical due to the onset of low-field loss.

## X. Conclusions

This paper has given the theory of quarter-wave coupled 3-port junctions in the case of $n=1$, 2, and 3. It applies to devices for which $s_0=-1$ at the terminals of the junction. This configuration includes the stripline one containing a simple dielectric or unmagnetized ferrite disk. It has also been shown that the theory of quarter-wave coupled junction circulators is closely related to the theory of reciprocal junctions with similar matching networks.

## References

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